

## Zero-Field Splitting, Field-Dependent Magnetization of Mixed-Valent $S = 3/2$ Diruthenium(II,III) Tetracarboxylates

William W. Shum, Yi Liao, and Joel S. Miller\*

Department of Chemistry, University of Utah, 315 South 1400 East Room 2124, Salt Lake City, Utah 84112-0850

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The 2 K field-dependent magnetization,  $M(H)$ , of  $S = 3/2$   $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  was studied.  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  exhibits an unusually low magnetization with respect to that predicted by the classical Brillouin function. This reduced value is a consequence of the large anisotropy arising from the large zero-field splitting (ZFS),  $D$  ( $+63 \pm 11 \text{ cm}^{-1}$ ), of the  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  cation, which alters the energy levels with respect to the isotropic energy levels used to derive the Brillouin function. Analytical expressions for the parallel and perpendicular components of  $M(H)$  that include zero-field splitting (ZFS),  $D$ , and interdimer coupling,  $\theta$ , are presented for  $S = 3/2$ . The expression was derived from second-order perturbation theory for  $|D| \gg g\mu_B H$ . The experimental data fit very well with  $g = 2.24 \pm 0.01$ ,  $D = +69.5 \text{ cm}^{-1}$  ( $D/k_B = +100 \text{ K}$ ), and  $0 > \theta > -0.6 \text{ K}$  indicative of very weak interdimer interactions for both  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]\text{Cl}$  and  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]_3[\text{Co}^{\text{III}}(\text{CN})_6]$ .

### Introduction

Field-dependent magnetization,  $M(H)$ , studies are frequently relied upon to ascertain the spin state of a paramagnetic site via fitting the data to the Brillouin function, as was first reported by Henry.<sup>1</sup> This is best established for isolated paramagnetic centers that do not have contributions to the magnetization from orbital angular momentum, spin–orbit coupling, and/or zero-field splitting, as analytical expressions that include these contributions have not been reported. Nonetheless, numerical methods have been developed principally to identify isolated paramagnetic centers present in some proteins.<sup>2</sup> Likewise, to understand the magnetic couplings (ferro- or antiferromagnetic) and the ground state of molecule-based magnets,  $M(H)$  studies are important. In particular we sought to identify the nature of the coupling present in  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]_3[\text{Cr}^{\text{III}}(\text{CN})_6]$  ( $T_c = 33 \text{ K}$ ). This magnet possesses  $S = 3/2$   $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  and  $S = 3/2$   $[\text{Cr}^{\text{III}}(\text{CN})_6]^{3-}$  spin sites;<sup>3</sup> however, octahedral  $\text{Cr}^{\text{III}}$  is well modeled by the Brillouin function, and  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  is not.

The physical properties of the mixed-valent,  $D_{4h}$   $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  have been extensively studied. This cation has a  $\sigma^2\pi^4\delta^2\delta^*1\pi^*2$   $S = 3/2$  valence electronic configuration<sup>4,5</sup> with spins fully delocalized between the two ruthenium centers. However,  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]^+$  has an unusually large zero-field splitting (ZFS),  $D$  ( $+63 \pm 11 \text{ cm}^{-1}$ ;  $D/k_B = 90.6 \pm 15.8 \text{ K}$ ).<sup>5–7</sup>  $[\text{Ru}^{\text{II/III}}_2(\text{OAc})_4]_3[\text{Cr}^{\text{III}}(\text{CN})_6]$  has  $D = 69.4 \text{ cm}^{-1}$  ( $D/k_B = 100 \text{ K}$ ); hence, at low temperature ( $\leq T_c$ ) only the  $m_s = 1/2$  state is significantly populated, complicating the analysis of the field-dependent magnetization,  $M(H)$  including an anomalous hysteresis loop.<sup>3</sup> Due to the presence of zero-field splitting, the Brillouin function cannot be used to model the  $M(H)$  data. Nonetheless, there are analytical models for anisotropic temperature-dependent magnetization,  $M(T,D)$ ,<sup>6a</sup> and herein we extend the methodology used to derive analytical expressions for  $M(T,D)$  to derive expressions for  $M(H,D)$ , and the derived expressions are used to fit the observed  $M(H)$  data for  $[\text{Ru}_2(\text{OAc})_4]\text{Cl}$  and  $[\text{Ru}_2(\text{OAc})_4]_3[\text{Co}(\text{CN})_6]$  with excellent agreement.

### Experimental Section

$[\text{Ru}_2(\text{OAc})_4]\text{Cl}$ , **1**, and  $[\text{Ru}_2(\text{OAc})_4]_3[\text{Co}(\text{CN})_6]$ , **2**, were prepared as previously described.<sup>3</sup> Field-dependent magnetization measurements were carried out on either a Quantum Design MPMS-5XL SQUID magnetometer from 0 to 5 T or a Quantum Design PPMS Model 9 T susceptometer from 0 to 7.4 T at 2 K as previously described.<sup>8</sup>

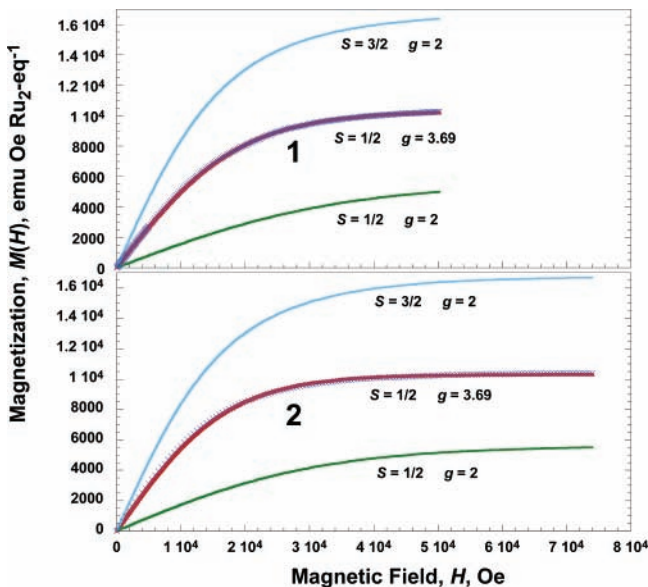
### Results and Discussion

At 2 K the  $M(H)$  of **1** and **2** were observed to be 10 270 and 10 262  $\text{emu}\cdot\text{Oe}$   $\text{Ru}_2\text{-eq}^{-1}$  at 5 T, respectively (Figure 1). These values are lower than predicted from the Brillouin function, eq 1 for  $S = 3/2$  (i.e., 16 755  $\text{emu}\cdot\text{Oe}$   $\text{mol}^{-1}$ ) due, as discussed above, to the extremely large ZFS of  $[\text{Ru}_2(\text{OAc})_4]^+$ <sup>5–7</sup> that depopulates the  $m_s = 3/2$  energy level at 2 K.<sup>9</sup> Thus, the only populated state is  $m_s = 1/2$ . This is in contrast to a 1:1 state occupation for the  $m_s = 3/2$  and  $m_s = 1/2$  states when the system is isotropic, i.e.,  $D = 0$ . Hence, data were fit to the Brillouin function for  $S = 1/2$  that includes a term to account for intradimer interactions  $\theta$ .<sup>10</sup>

$$M(H,\theta)_{\text{Brillouin}} =$$

$$Ng\mu_B S \left[ (2S + 1) \coth \left( \frac{g\mu_B SH}{k_B(T - \theta)} \frac{2S + 1}{2S} \right) - \coth \left( \frac{g\mu_B SH}{k_B(T - \theta)} \frac{1}{2S} \right) \right] \frac{1}{2S} \quad (1)$$

where  $N$  is Avogadro's number,  $g$  is the Landé factor,  $\mu_B$  is the Bohr magneton,  $S$  is the spin quantum number, and  $H$  is the magnetic field, with  $g = 3.69$ ,  $\theta = -0.35 \text{ K}$  for **1**, and  $g = 3.69$ ,  $\theta = -0.12 \text{ K}$  and for **2** (Figure 1). The small  $\theta$  values indicate very weak intradimer antiferromagnetic coupling. Weaker coupling is expected via the three diamagnetic five-atom  $-\text{NCCo}^{\text{III}}\text{CN}-$  bridges for **2** with respect to the diamagnetic single-atom  $\text{Cl}^-$  bridge for **1**, as observed.



**Figure 1.**  $S = 1/2$  Brillouin function fit, eq 1, with the data of **1** ( $S = 1/2$ ,  $g = 3.69$ ,  $\theta = -0.35$  K), and **2** ( $S = 1/2$ ,  $g = 3.69$ ,  $\theta = -0.12$  K). The calculated  $M(H)$  for  $g = 2$ ,  $S = 1/2$ , and  $S = 3/2$  from eq 1 are shown for comparison. The observed data are plotted as  $x$ 's.

The large unphysical 3.69  $g$ -value fit to the Brillouin function emphasizes the inappropriateness of eq 1, which is attributed to the different splitting magnitudes of the  $m_s = 1/2$  states arising from the ZFS. The Zeeman splitting used to derive the Brillouin function is  $E = \pm g\mu_B H/2$  for  $S = 1/2$  but is not valid due to ZFS. Taking into account the anisotropy arising from the ZFS, the splitting for  $m_s = 1/2$  is  $E_{||} = \pm g_{||}\mu_B H/2$ , and  $E_{\perp} = \pm g_{\perp}\mu_B H/2 - 3g_{\perp}^2\mu_B^2 H_{\perp}^2/(8D)$ ,<sup>11a</sup> Figure 2. Since  $\mu_B = 9.274 \times 10^{-24}$  J T<sup>-1</sup> is small, the latter term for  $E_{\perp}$  is negligible. Hence, the perpendicular splitting is twice that of the parallel for  $M(H,D)$ , thus  $g_{M(H,D)\perp} = 2g_{M(H,D)\parallel}$ . Given that the Brillouin function is isotropic,  $g_{\text{Brillouin}} = g_{\text{Brillouin}\perp} = g_{\text{Brillouin}\parallel}$ . The average  $g$  value for  $M(H,D)$  is  $g_{M(H,D)}$  where  $g_{M(H,D)} = (2g_{M(H,D)\perp} + g_{M(H,D)\parallel})/3 = (5g_{M(H,D)\parallel})/3$ .  $g_{||}$  for the Brillouin and  $M(H,D)$  expressions are the same, i.e.,  $g_{M(H,D)\parallel} = g_{\text{Brillouin}\parallel} = g_{\text{Brillouin}} = \pm g\mu_B H/2$ . Thus,  $g_{M(H,D)} = (5/3)g_{\text{Brillouin}} = 2.2$ ; hence, an alternative model is required. Consequently, we sought to fit the data with an analytical expression for  $M(H,D,\theta)$ .

Analytical expressions for  $M(H,D,\theta)$  are not readily available, but numerical methods have been used.<sup>12a</sup> The  $M(H,D)$  calculation by numerical methods takes into account the integration over all space. The integration ensures that all orientations of the sample are included,<sup>12b</sup> but analytical expressions for this have not been reported. Nonetheless, the structure of diruthenium complex is 3-D body centered, interpenetrating cubic lattice, and the orientation of the crystal at all directions are equivalent, and consequently an analytical expression for  $M(H,D,\theta)$  was derived.

**Analytical Expression for  $M(H,D,\theta)$ .** [Ru<sub>2</sub>(OAc)<sub>4</sub>]<sup>+</sup> possesses an <sup>4</sup>B<sub>2u</sub> ground state and <sup>2</sup>A<sub>1u</sub>, <sup>2</sup>A<sub>2u</sub>, <sup>2</sup>B<sub>2u</sub>, <sup>2</sup>B<sub>1u</sub> excited states; however, the excited states do not contribute to the paramagnetism.<sup>5b</sup> Thus, the excited states are neglected. In the case of isotropic  $S = 3/2$ , both the  $m_s = \pm 3/2$  and  $\pm 1/2$  energy levels are essentially equally populated. These states, however, are not evenly populated due to the ZFS,  $D$ , arising from the tetragonal distortion, and the larger the  $|D|$ , the greater the difference in the population of the states, especially for  $T \approx |D|$ . The ZFS Hamiltonian,  $\hat{H}_{\text{ZFS}}$ , in an octahedral crystal field with Zeeman effect is used to describe this phenomenon.

$$\hat{H}_{\text{ZFS}} = g\mu_B \hat{S} \cdot \underline{H} + D \left[ \hat{S}_z^2 - \frac{S(S+1)}{3} \right] + E (\hat{S}_x^2 - \hat{S}_y^2) \quad (2)$$

where  $D$  is the axial ZFS tensor,  $E$  is the rhombic ZFS tensor,  $S_z$  is the spin at parallel direction with respect to  $H$ , while  $S_x$  and  $S_y$  are the spins perpendicular with respect to  $H$ . Since there is no rhombic distortion in the system (i.e.,  $E = 0$ ),<sup>6</sup> the Hamiltonian reduces to:

$$\hat{H}_{\text{ZFS}} = g\mu_B \hat{S}_z \cdot H_z + g_x \mu_B \hat{S}_x \cdot H_x + g_y \mu_B \hat{S}_y \cdot H_y + D \left[ \hat{S}_z^2 - \frac{S(S+1)}{3} \right] \quad (3)$$

and the energy of the ZFS Hamiltonian can be expressed by the secular determinant as:

$$\begin{vmatrix} \langle 3/2 | & \langle 1-3/2 | & \langle 1/2 | & \langle -1/2 | \\ \langle 3/2 | & 2D + \frac{3}{2}g\mu_B H_z & 0 & \frac{\sqrt{3}}{2}(g\mu_B H_x + ig\mu_B H_y) \\ \langle -3/2 | & 0 & 2D - \frac{3}{2}g\mu_B H_z & 0 \\ \langle 1/2 | & \frac{\sqrt{3}}{2}(g\mu_B H_x - ig\mu_B H_y) & 0 & \frac{1}{2}g\mu_B H_z \\ \langle -1/2 | & 0 & \frac{\sqrt{3}}{2}(g\mu_B H_x - ig\mu_B H_y) & -\frac{1}{2}g\mu_B H_z \end{vmatrix} = E_{\text{ZFS}} \quad (4)$$

The anisotropic magnetization function,  $M(H,D,\theta)$ , for  $S = 3/2$  is derived from the ZFS Hamiltonian, eq 3, and magnetization equation. We introduce the Weiss constant,  $\theta$ , to account for weak intermolecular coupling.<sup>11b</sup>

$$M(H,D,\theta)_{||} = N \left[ 3g\mu_B \sinh \left( \frac{3}{2} \frac{g\mu_B H}{k_B(T-\theta)} \right) \exp(-2D/k_B T) + g\mu_B \sinh \left( \frac{g\mu_B H}{2k_B(T-\theta)} \right) \right] \left[ 2 \cosh \left( \frac{3}{2} \frac{g\mu_B H}{k_B(T-\theta)} \right) \times \exp(-2D/k_B T) + 2 \cosh \left( \frac{g\mu_B H}{2k_B(T-\theta)} \right) \right] \quad (5a)$$

$$M(H,D,\theta)_{\perp} = N \left[ \frac{-3}{2D} g^2 \mu_B^2 H \exp \left( \frac{-2D - \left[ \frac{3}{8D} g^2 \mu_B^2 H^2 \right]}{k_B T} \right) + \frac{3}{2D} g^2 \mu_B^2 H \cosh \left( \frac{-g\mu_B H}{k_B(T-\theta)} \right) - 2g\mu_B \sinh \left( \frac{-g\mu_B H}{k_B(T-\theta)} \right) \right] \left[ 2 \exp \left( \frac{-2D - \left[ \frac{3}{8D} g^2 \mu_B^2 H^2 \right]}{k_B T} \right) + 2 \cosh \left( \frac{-g\mu_B H}{k_B(T-\theta)} \right) \right] \quad (5b)$$

$$M(H,D,\theta)_{\text{AVERAGE}} = \frac{M(H)_{||} + 2M(H)_{\perp}}{3} \quad (5c)$$

Indeed, using eq 5,  $M(H,D,\theta)$  gave the best fit for **1** with  $g = 2.253$ ,  $D = 69.4$  cm<sup>-1</sup> ( $D/k_B = 100$  K), and  $\theta = -0.56$  K, with a  $\chi^2$  agreement factor<sup>13</sup> =  $\sum (M_{\text{observed}} - M_{\text{calc}})^2 / M_{\text{observed}}^2 = 1.0046$ . The best fit for **2** with  $g = 2.235$ ,  $D = 69.4$  cm<sup>-1</sup>

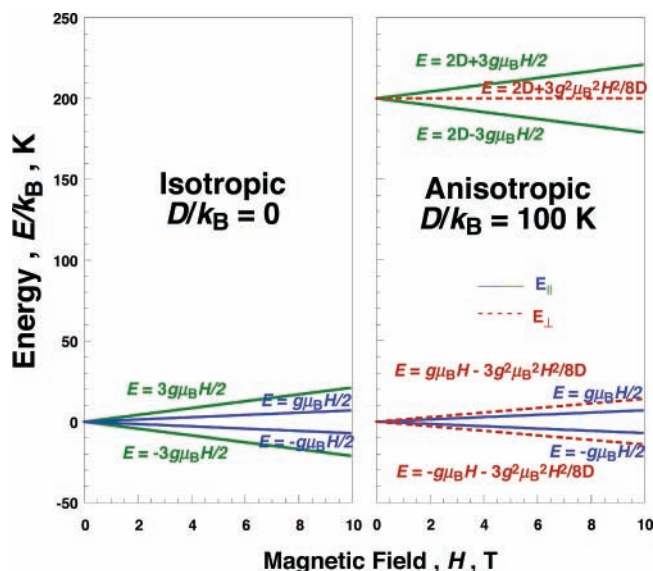


Figure 2. Energy spectra of  $S = 3/2$  energy levels with Zeeman effect, and isotropic and anisotropic for  $D = 69.4 \text{ cm}^{-1}$  ( $D/k_B = 100 \text{ K}$ ).

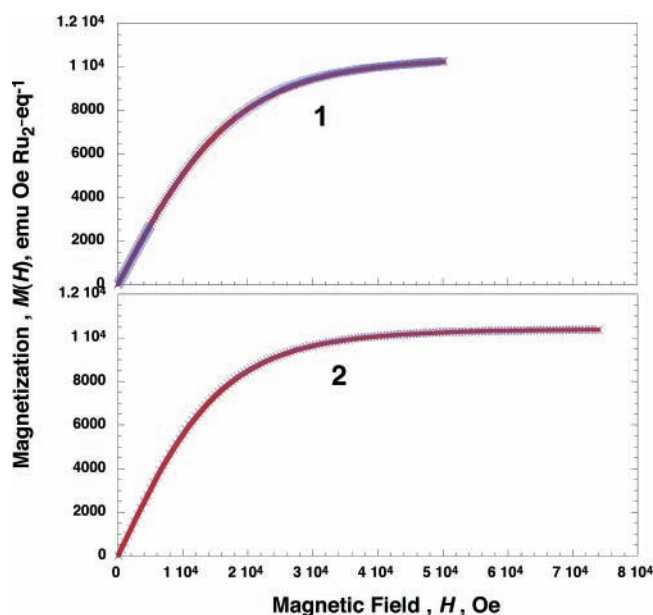


Figure 3. Observed  $M(H, D, \theta)$  ( $\times$ ) for **1** ( $g = 2.253$ ,  $D = 69.4 \text{ cm}^{-1}$  ( $D/k_B = 100 \text{ K}$ ), and  $\theta = -0.56 \text{ K}$ ) and **2** ( $g = 2.235$ ,  $D = 69.4 \text{ cm}^{-1}$  ( $D/k_B = 100 \text{ K}$ ),  $\theta = -0.24 \text{ K}$ ), and their fits to eq 5.

( $D/k_B = 100 \text{ K}$ ),  $\theta = -0.24 \text{ K}$  ( $\chi^2 = 0.93$ ) (Figure 3). Again, the small  $\theta$  values indicate very weak intradimer antiferromagnetic coupling.

The observed magnetizations  $10,270$  and  $10,262 \text{ emu}\cdot\text{Oe Ru}_2\text{-eq}^{-1}$  at  $5 \text{ T}$  for **1** and **2**, respectively, are consistent with only the  $m_s = 1/2$  energy level being populated. The observed magnetization is the first plateau, and it should eventually rise to about  $18,900 \text{ emu}\cdot\text{Oe Ru}_2\text{-eq}^{-1}$  when saturation occurs. This saturation magnetization is predicted to be the same value as that predicted by the Brillouin function, i.e.,  $18,880$  and  $18,725 \text{ emu}\cdot\text{Oe Ru}_2\text{-eq}^{-1}$  for **1** and **2**, respectively.

Since the anisotropic magnetization function is derived from second-order degenerate perturbation,  $|D| \gg g\mu_B H$  was assumed. For large applied magnetic fields,  $D \approx g\mu_B H$ ; hence, second-order perturbation is not valid. Consequently, there will be energy crossing when  $D \approx g\mu_B H$ , but to fully understand and

to predict this energy-crossing phenomena, which results in the magnetization steps, an exact solution of the Hamiltonian, or higher-order perturbations, is required. This is a focus of ongoing studies, which will predict both energy-crossing and noncrossing effects. The noncrossing energy is due to the noncrossing rule, in which energy from spins that possess the same symmetry does not cross, and as a consequence, the energy-level mixing should occur.

## Conclusion

Extension of the classical Brillouin function,  $M(H)$ , to include zero-field splitting (ZFS),  $D$ , [ $|D| \gg g\mu_B H$ ] [and an intermolecular interaction ( $\theta$ )], to a general analytical expression for the anisotropic magnetization function,  $M(H, D, \theta)$ , has been derived. This equation describes the unusually low values of the observed magnetization for  $[\text{Ru}^{\text{III}}_2(\text{OAc})_4]^+$ . Deviations from the classical Brillouin function are a consequence of differing energy levels with respect to the isotropic energy levels used to derive the Brillouin function. However, further theoretical studies and high-field experiments will enable the understanding of the spin behavior upon saturation and energy-level crossover for materials with zero-field splitting.

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